CCFEA. Credit and Default Modeling. Unit 7: Counterparty Risk with Stochastic Dynamical Models: Impact of Volatilities and Correlations

Damiano Brigo

www.damianobrigo.it

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Agenda

1. Common questions and Introduction
2. The mechanics of counterparty risk
   - General formula, Symmetry vs Asymmetry
   - Contingent Credit Default Swap
3. Our modeling approach
   - Credit Modeling Assumptions
   - Modeling the credit part
   - Modeling the underlying
4. Three applications: Rates, Commodities and Credit
   - Cases from three asset classes
5. Conclusions and References

Some common questions 1

Q What is counterparty risk in general?

A *The risk taken on by an entity entering an OTC contract with a counterparty having a relevant default probability. As such, the counterparty might not respect its payment obligations.*

Q When is valuation of counterparty risk symmetric?

A *When we include the possibility that also the entity computing the counterparty risk adjustment may default, besides the counterparty itself.*

Q When is valuation of counterparty risk asymmetric?

A *When the entity computing the counterparty risk adjustment considers itself default-free, and only the counterparty may default.*

Q Which one is computed usually for valuation adjustments?

A *The asymmetric one.*

Some common questions 2

Q What impacts counterparty risk?

A *The OTC contract's underlying volatility, the correlation between the underlying and default of the counterparty, and the counterparty credit spreads volatility.*

Q Is it model dependent?

A *It is.*

Q What about *wrong way risk*?

A *The amplified risk when the reference underlying and the counterparty are strongly correlated in the wrong direction.*

Existing approaches for the Asymmetric Case

Capital Adequacy based approach

- Obtain estimates of expected exposures for the portfolio NPV at different maturities
- Buy default protection on the counterparty through Credit Default Swaps on those maturities with notionals following the expected exposures.

Problems

- May ignore risk premiums in the underlying portfolio's factors
- Ignores correlation structure between counterparty default and portfolio's risk factors
- Models wrong way risk inaccurately through rough coefficients

General Notation

- We will call “investor" the party interested in the counterparty adjustment. This is denoted by “0"
- We will call “counterparty" the party with whom the investor is trading, and whose default may affect negatively the investor. This is denoted by “2" or “C".
- “1" will be used to denote the underlying name/risk factor(s) of the contract

The mechanics of Evaluating asymmetric counterparty risk

original payoff of the instrument

all cash flows before default
⊕ recovery of the residual NPV at default if positive
⊖ Total residual NPV at default if negative

General Formulation under Asymmetry

If one writes the cash flows corresponding to the above diagram and then simplifies the indicators, one obtains the fundamental formula for the valuation of counterparty risk when the investor is default free:

$$\mathrm{E}_t\left\{\Pi^D(t,T)\right\} = \mathrm{E}_t\left\{\Pi(t,T)\right\} -$$
$$\mathrm{LGD}\cdot\mathrm{E}_t\left\{\mathbf{1}(t<\tau_C\leqslant T)\cdot D(t,\tau_C)\cdot\left[\mathrm{NPV}(\tau_C)\right]^+\right\}$$

- First term : Value without counterparty risk.
- Second term : Counterparty risk adjustment.
- $\mathrm{NPV}(\tau_C) = \mathrm{E}_{\tau_C}\left[\Pi(\tau_C,T)\right]$ is the value of the transaction on the counterparty default date. LGD = 1 - REC_counterparty.

What we can observe

- Including counterparty risk in the valuation of an otherwise default-free derivative $\Longrightarrow$ credit (hybrid) derivative.

- The inclusion of counterparty risk adds a level of optionality to the payoff.
  In particular, model independent products become model dependent also in the underlying market.
  $\Longrightarrow$ **Counterparty Risk analysis incorporates an opinion about the underlying market dynamics and volatility.**

Including the investor default or not?

Often the investor, when computing a counterparty risk adjustment, considers itself to be default-free. This can be either a unrealistic assumption or an approximation for the case when the counterparty has a much higher default probability than the investor.

If this assumption is made counterparty risk is asymmetric: if "2" were to consider "0" as counterparty for a moment and computed the counterparty risk adjustment, this would not be the opposite of the one computed by "0" in the straight case.

Also, the total NPV including counterparty risk is similarly asymmetric, in that the total value of the position to "0" is not the opposite of the total value of the position to "2".

Including the investor default or not?

We get back symmetry if we allow for default of the investor in computing counterparty risk. This also results in an adjustment that is cheaper to the counterparty “2".

The counterparty “2" may then be willing to ask the investor “0" to include the investor default event into the model, when the Counterparty risk adjustment is computed by the investor

In most of the presentation examples we deal with asymmetric risk, i.e. we assume the investor to be default free.

The case of symmetric counterparty risk

Suppose now that we allow for both parties to default.
Counterparty risk adjustment allowing for default of “0"?
“0" : the investor; “2": the counterparty;
(“1": the underlying name/risk factor of the contract).
τ_0, τ_2: default times of “0" and “2". T: final maturity
We consider the following events, forming a partition

Four events ordering the default times

$$
\begin{aligned}
A &= \{\tau_0 \le \tau_2 \le T\} \qquad E = \{T \le \tau_0 \le \tau_2\} \\
B &= \{\tau_0 \le T \le \tau_2\} \qquad F = \{T \le \tau_2 \le \tau_0\} \\
C &= \{\tau_2 \le \tau_0 \le T\} \\
D &= \{\tau_2 \le T \le \tau_0\}
\end{aligned}
$$

The case of symmetric counterparty risk

$$
\begin{aligned}
\mathrm{E}_t \left\{\Pi^D(t,T)\right\} &= \mathrm{E}_t\left\{\Pi(t,T)\right\} \\
&+ \mathrm{LGD}_0 \cdot \mathrm{E}_t \left\{ \mathbf{1}(A \cup B) \cdot D(t,\tau_0) \cdot \left[-\mathrm{NPV}(\tau_0)\right]^+ \right\} \\
&- \mathrm{LGD}_2 \cdot \mathrm{E}_t \left\{ \mathbf{1}(C \cup D) \cdot D(t,\tau_2) \cdot \left[\mathrm{NPV}(\tau_2)\right]^+ \right\}
\end{aligned}
$$

- This formula can be proved writing the proper cash flows in each senario and adding them up + simplifying.
- 2nd term : Counterparty risk adj due to scenarios $\tau_0 < \tau_2$.
- 3d term : Counterparty risk adj due to scenarios $\tau_2 < \tau_0$.
- If computed from the opposite point of view of “2" having counterparty “0", the adjustment is the opposite. Symmetry.

The case of symmetric counterparty risk

When allowing for the investor to default: symmetry

- One more term with respect to the asymmetric case.
- depending on credit spreads and correlations, the adjustment to be subtracted can now be either positive or negative. In the asymmetric case it can only be positive.
- Ignoring the symmetry is clearly more expensive for the counterparty and cheaper for the investor.
- Some counterparties therefore may request the investor to include its own default into the valuation
- We assume the asymmetric case from now on

A useful derivative: Contingent CDS (CCDS)

Definition

Similar to a CDS but when the reference credit defaults at τ, the protection seller pays protection on a notional that is not fixed but given by the NPV of a reference Portfolio Π at that time if positive. This amount is:
$(\mathbb{E}_{\tau_C}\Pi(\tau_C, T))^+$, minus a recovery Rec fraction of it.

CCDS default leg payoff = asymmetric counterparty risk adj

The payoff of the default leg of a Contingent CDS is exactly

$$(1 - \text{Rec})\mathbf{1}_{\{(t<\tau_C<T)\}}D(t, \tau_C)(\mathbb{E}_{\tau_C}\Pi(\tau_C, T))^+$$

General Remarks on CCDS

Financial Times, April 10, 2008:

> *[...]Rudimentary and idiosyncratic versions of these so-called CCDS have existed for five years, but they have been rarely traded due to high costs, low liquidity and limited scope.*
>
> ***But now there are high hopes that a revamped version of CCDS, which will bear the formal blessing of the International Swaps and Derivatives Association, will be more successful when it is released in two to three months' time.***

General Remarks on CCDS

[...] Counterparty risk has become a particular concern in the markets for interest rate, currency, and commodity swaps - because these trades are not always backed by collateral.

Many of these institutions - such as hedge funds and companies that do not issue debt - are beyond the scope of cheaper and more liquid hedging tools such as normal CDS. The new CCDS was developed to target these institutions.

Being the two payoffs equivalent, the counterparty risk adjustment valuation will hold as well for the default leg of a CCDS.

Methodology

1. Assumption: The *investor* enters a transaction with a *counterparty* and the investor considers itself default free. Note : All the payoffs seen from the point of view of the *investor*.
2. We model and calibrate the default time of the *counterparty* using a stochastic intensity default model.
3. We model the transaction underlying and estimate the deal NPV at default.
4. We allow for the counterparty default time and the contract underlying to be correlated.

Counterparty default model: CIR++ stochastic intensity

The model for the counterparty instantaneous credit spread:

$$\lambda(t) = y(t) + \psi(t;\beta)$$

Remarks:

1. $y(t)$ is a CIR process with possible jumps
   $\mathrm{d}y = \kappa(\mu - y)\mathrm{d}t + \nu\sqrt{y}\mathrm{d}W_y + \mathrm{d}J$
2. $\psi(t;\beta)$ is the shift that matches a given CDS curve
3. In CDS calibration we assume deterministic interest rates.
4. Calibration : Fitting model survival probabilities to survival probabilities stripped from counterparty CDS quotes

Approximation: Default Bucketing

General Formulation

1. Model (underlying) to estimate the NPV of the transaction.
2. Simulations are run allowing for correlation between the credit and underlying models, to determine the counterparty default time and the underlying deal NPV respectively.

Approximated Formulation under default bucketing

$$\mathbb{E}_0 \Pi^D(0,T) := \mathbb{E}_0 \Pi(0,T)$$

$$-\text{L}_{\text{GD}} \sum_{j=1}^{b} \mathbb{E}_0[1\{\tau \in (T_{j-1}, T_j]\} \; D(0,T_j)(\mathbb{E}_{T_j}\Pi(T_j,T))^+]$$

1. In this formulation defaults are bucketed but we still need a joint model for τ and the underlying Π including their correlation.
2. Option model for Π is implicitly needed in τ scenarios.

Approximation: Default Bucketing and Independence

Approximated Formulation under independence (and 0 correlation)

$$\mathbb{E}_0 \Pi^D(0,T) := \mathbb{E}_0 \Pi(0,T)$$

$$-\mathrm{L{\scriptstyle GD}} \sum_{j=1}^{b} \boxed{\mathbb{Q}\{\tau \in (T_{j-1}, T_j]\}} \, \mathbb{E}_0[D(0,T_j)(\mathbb{E}_{T_j}\Pi(T_j,T))^+]$$

1. In this formulation defaults are bucketed and only survival probabilities are needed (no default model).
2. Option model is STILL needed for the underlying of Π.

Three cases: Interest Rates, Credit and Commodities

We now examine three specific cases of underlying contracts:

- Interest Rate Swaps and Derivatives Portfolios
- Commodities swaps (Oil)
- Credit: CDS on a reference credit

Interest Rates Swap Case

Formulation for IRS under independence (no correlation)

$$\mathrm{IRS}^D(t,K) = \mathrm{IRS}(t.K)$$

$$-\textsc{Lgd} \sum_{i=a+1}^{b-1} \mathbb{Q}\{\tau \in (T_{i-1}, T_i]\}\, \textsc{Swaption}_{i,b}(t; K, S_{i,b}(t), \sigma_{i,b})$$

Modeling Approach with corr.

Gaussian two-factor short-rate $r(t)$ model:
$r(t) = x(t) + z(t) + \varphi(t;\alpha), r(0) = r_0$

$$dx(t) = -ax(t)dt + \sigma dW_x$$
$$dz(t) = -bz(t)dt + \eta dW_z$$

$$dW_x dW_z = \rho_{x,z} dt$$

$$\alpha = [r_0, a, b, \sigma, \eta, \rho_{1,2}]$$

$$dW_x dW_y = \rho_{x,y} dt, dW_z dW_y = \rho_{z,y} dt$$

Calibration

- The function $\varphi(\cdot;\alpha)$ is deterministic and is used to calibrate the initial curve observed in the market.
- We use swaptions and zero curve data to calibrate the model.
- The r factors x and z and the intensity are taken to be correlated.

Interest Rates Swap Case

Total Correlation Counterparty default / rates

$$\bar{\rho} = \text{Corr}(dr_t, d\lambda_t) = \frac{\sigma \rho_{x,y} + \eta \rho_{z,y}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{x,z}}\sqrt{1 + \frac{2\beta\gamma^2}{\nu^2 y_t}}}.$$

where β is the intensity of arrival of λ jumps and γ is the mean of the exponentially distributed jump sizes.

Without jumps ($\beta = 0$)

$$\bar{\rho} = \text{Corr}(dr_t, d\lambda_t) = \frac{\sigma \rho_{x,y} + \eta \rho_{z,y}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{x,z}}}.$$

IRS: Case Study

1) Single Interest Rate Swaps (IRS)

At-the-money fix-receiver forward interest-rate-swap (IRS) paying on the EUR market.

The IRS's fixed legs pay annually a 30E/360 strike rate, while the floating legs pay LIBOR twice per year.

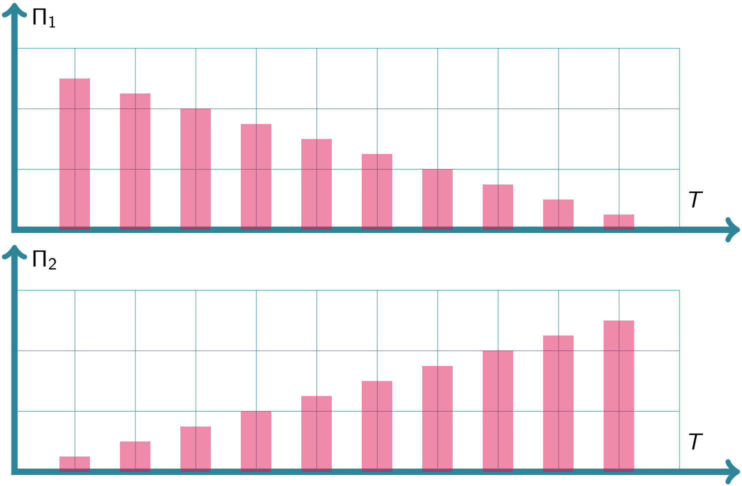
2) Netted portfolios of IRS.

- Portfolios of at-the-money IRS either with different starting dates or with different maturities.

1. ($\Pi 1$) annually spaced dates $\{T_i : i = 0 \dots N\}$, T_0 two business days from trade date; portfolio of swaps maturing at each T_i, with $i > 0$, all starting at T_0.
2. ($\Pi 2$) portfolio of swaps starting at each T_i all maturing at T_N.

Can also do exotics (Ratchets, CMS spreads, Bermudan)

IRS Case Study: Payment schedules



IRS Results

Counterparty risk price for netted receiver IRS portfolios Π1 and Π2 and simple IRS (maturity 10Y). Every IRS, constituting the portfolios, has unit notional and is at equilibrium. Prices are in bps.

λ	correlation $\bar{\rho}$	Π1	Π2	IRS
3%	-1	-140	-294	-36
	0	-84	-190	-22
	1	-47	-115	-13
5%	-1	-181	-377	-46
	0	-132	-290	-34
	1	-99	-227	-26
7%	-1	-218	-447	-54
	0	-173	-369	-44
	1	-143	-316	-37

Compare with "Basel 2" deduced adjustments

Basel 2 models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case.

Is this confirmed by our model?

$$(140 - 84)/84 \approx 66\% > 40\%$$

$$(54 - 44)/44 \approx 23\% < 40\%$$

So this really depends on the portfolio and on the situation.

Payer vs Receiver

- Counterparty Risk (CR) has a relevant impact on interest-rate payoffs prices and, in turn, correlation between interest-rates and default (intensity) has a relevant impact on the CR adjustment.
- The (positive) CR adjustment to be subtracted from the default free price **decreases with correlation for receiver payoffs**. Natural: If default intensities increase, with high positive correlation their correlated interest rates will increase more than with low correlation, and thus a receiver swaption embedded in the adjustment decreases more, reducing the adjustment.
- The adjustment for payer payoffs increases with correlation.

Further Stylized Facts

- As the default probability implied by the counterparty CDS increases, the size of the adjustment due to counterparty risk increases as well, but the impact of correlation on it decreases.
- This is financially reasonable: Given large default probabilities for the counterparty, fine details on the dynamics such as the correlation with interest rates become less relevant, everything being wiped out by massive defaults anyway.
- **The conclusion is that we should take into account interest-rate/ default correlation in valuing CR interest-rate payoffs.**

Exotics

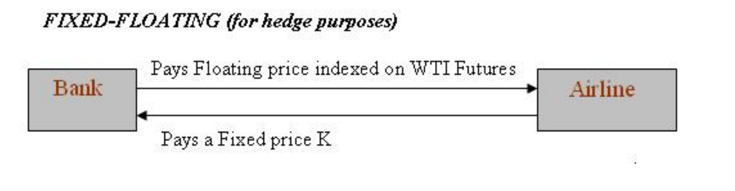
For examples on exotics, including Bermudan Swaptions and CMS spread Options, see

Paper with Exotics

Brigo, D., and Pallavicini, A. (2007). Counterparty Risk under Correlation between Default and Interest Rates. In: Miller, J., Edelman, D., and Appleby, J. (Editors), Numerical Methods for Finance, Chapman Hall.
http://www.defaultrisk.com/pp_model143.htm

Commodities: Futures, Forwards and Swaps

- **Forward**: OTC contract to buy a commodity to be delivered at a maturity date T at a price specified today. The cash/commodity exchange happens at time T.
- **Future**: Listed Contract to buy a commodity to be delivered at a maturity date T. Each day between today and T margins are called and there are payments to adjust the position.
- **Commodity Swap: Oil Example**:



Commodities: Modeling Approach

Schwartz-Smith Model

$$ln(S_t) = x_t + l_t + \varphi(t)$$
$$dx_t = -kx_t dt + \sigma_x dW_x$$
$$dl_t = \mu dt + \sigma_l dW_l$$
$$dW_x \; dW_l = \rho_{x,l} dt$$

Variables

S_t: Spot oil price;
x_t, l_t: short and long term components of S_t;
This can be re-cast in a classic convenience yield model

Correlation with credit

$$dW_x \; dW_y = \rho_{x,y} dt,$$
$$dW_l \; dW_y = \rho_{l,y} dt$$

Calibration

φ: defined to exactly fit the oil forward curve.
Dynamic parameters k, μ, σ, ρ are calibrated to At the money implied volatilities on Futures options.

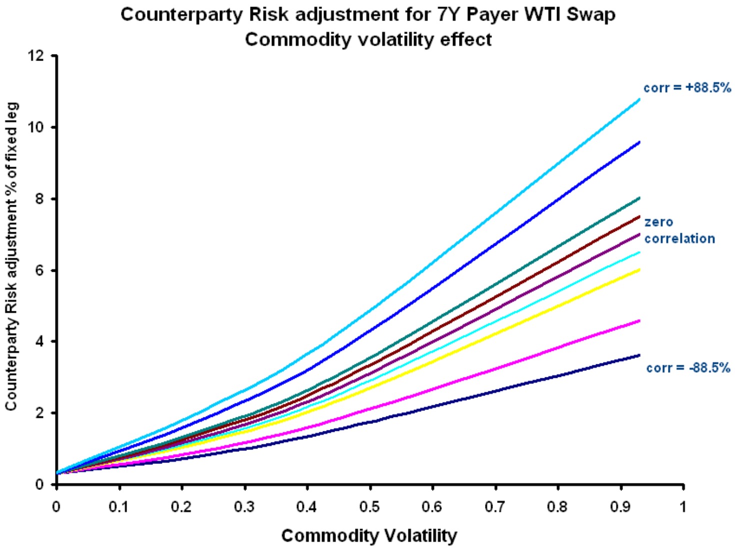
Commodities

Total correlation Commodities - Counterparty default

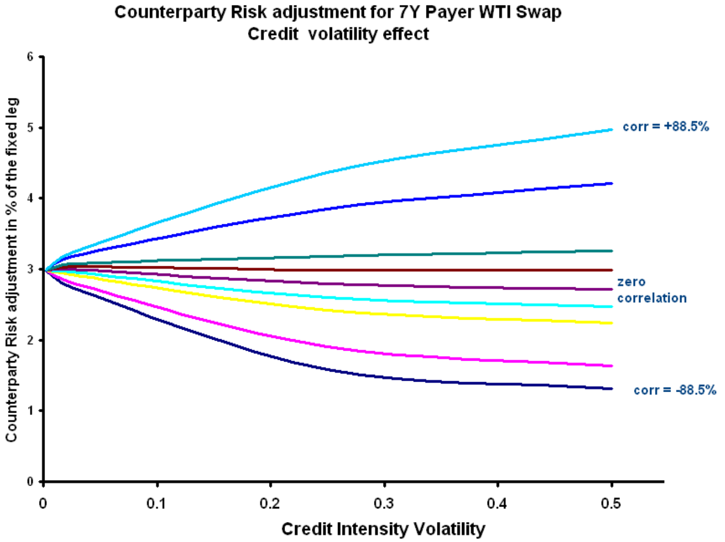
$$\bar{\rho} = \text{corr}(d\lambda_t,\ dS_t) = \frac{\sigma_x \rho_{x,y} + \sigma_L \rho_{L,y}}{\sqrt{\sigma_x^2 + \sigma_L^2 + 2\rho_{x,L}\sigma_x\sigma_L}}$$

We assumed no jumps in the intensity

Commodity Swap Results Overview : Commodity Volatility Effect



Commodity Swap Results Overview: Credit Volatility Effect



Commodity Swap Results[1] : Credit volatility effect

$\bar{\rho}$	intensity volatility ν_R	0.025	0.25	0.50
-88.5	Payer adj	2.742	1.584	1.307
	Receiver adj	1.878	2.546	3.066
-63.2	Payer adj	2.813	1.902	1.63
	Receiver adj	1.858	2.282	2.632
-25.3	Payer adj	2.92	2.419	2.238
	Receiver adj	1.813	1.911	2.0242
-12.6	Payer adj	2.96	2.602	2.471
	Receiver adj	1.802	1.792	1.863
0	Payer adj	2.999	2.79	2.719
	Receiver adj	1.79	1.676	1.691
+12.6	Payer adj	3.036	2.985	2.981
	Receiver adj	1.775	1.562	1.527
+25.3	Payer adj	3.071	3.184	3.258
	Receiver adj	1.758	1.45	1.371
+63.2	Payer adj	3.184	3.852	4.205
	Receiver adj	1.717	1.154	0.977
+88.5	Payer adj	3.229	4.368	4.973
	Receiver adj	1.664	0.988	0.798
Fixed Leg Price maturity 7Y: 7345.39 USD for a notional of 1 Barrel per Month				

[1] adjusment expressed as % of the fixed leg price

Commodity Swap Results[2] : Commodity volatility effect

$\bar{\rho}$	Commodity spot volatility σ_S	0.0005	0.232	0.46	0.93
-88.5	Payer adj	0.322	0.795	1.584	3.607
	Receiver adj	0	1.268	2.546	4.495
-63.2	Payer adj	0.322	0.94	1.902	4.577
	Receiver adj	0	1.165	2.282	4.137
-25.3	Payer adj	0.323	1.164	2.419	6.015
	Receiver adj	0	0.977	1.911	3.527
-12.6	Payer adj	0.323	1.246	2.602	6.508
	Receiver adj	0	0.917	1.792	3.325
0	Payer adj	0.324	1.332	2.79	6.999
	Receiver adj	0	0.857	1.676	3.115
+12.6	Payer adj	0.324	1.422	2.985	7.501
	Receiver adj	0	0.799	1.562	2.907
+25.3	Payer adj	0.324	1.516	3.184	8.011
	Receiver adj	0	0.742	1.45	2.702
+63.2	Payer adj	0.325	1.818	3.8525	9.581
	Receiver adj	0	0.573	1.154	2.107
+88.5	Payer adj	0.326	2.05	4.368	10.771
	Receiver adj	0	0.457	0.988	1.715

Fixed Leg Price maturity 7Y: 7345.39 USD for a notional of 1 Barrel per Month

[2] adjusment expressed as % of the fixed leg price

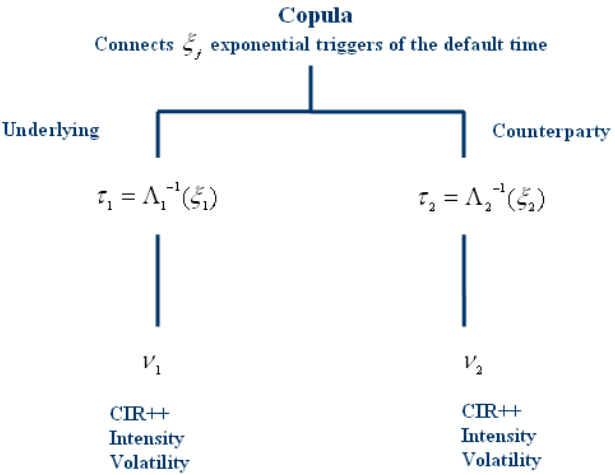
Credit (CDS)

- Model equations: ("1" = CDS underlying, "2" = counterparty)

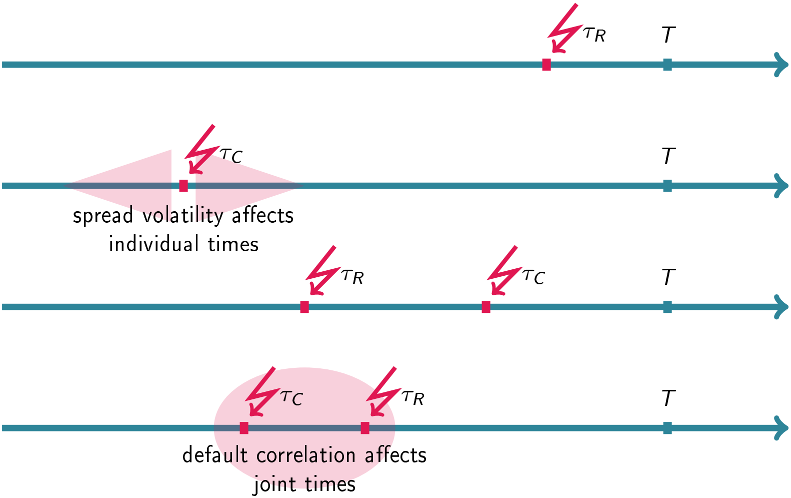
$$d\lambda_j(t) = k_j(\mu_j - \lambda_j(t))dt + \nu_j\sqrt{\lambda_j(t)}dZ_j(t), \quad j = 1, 2$$

- Cumulative intensities are defined as : $\Lambda(t) = \int_0^t \lambda(s)\mathrm{d}s$.
- Default times are $\tau_j = \Lambda_j^{-1}(\xi_j)$. Exponential triggers ξ_1 and ξ_2 are connected through a gaussian copula with correlation parameter ρ.
- In our approach, we take into account default correlation between default times τ_1 and τ_2 **and** credit spreads volatility $\nu_j, j = 1, 2$.
- Important: volatility can amplify default time uncertainty, while high correlation reduces conditional default time uncertainty.
  Taking into account ρ and $\nu \implies$ better representation of market information and behavior, especially for wrong way risk.

Credit (CDS) : Overview

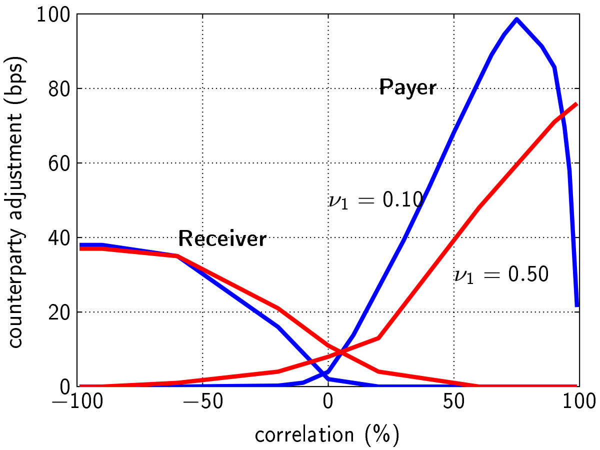


Credit (CDS) Correlation and Volatility Effects



Moderate counterparty spread $\nu_2 = 0.10$

ρ	Vol parameter ν_1	0.01	0.10	0.20	0.30	0.40	0.50
	CDS Implied vol	1.5%	15%	28%	37%	42%	42%
-99	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	40(2)	38(2)	39(2)	38(2)	36(1)	37(1)
-90	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	39(2)	38(2)	38(2)	38(2)	35(1)	37(2)
-60	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	1(0)
	Receiver adj	36(1)	35(1)	36(1)	36(1)	32(1)	35(1)
-20	Payer adj	0(0)	0(0)	1(0)	2(0)	3(0)	4(1)
	Receiver adj	16(1)	16(1)	17(1)	19(1)	18(1)	21(1)
0	Payer adj	3(0)	4(0)	5(0)	7(1)	7(1)	8(1)
	Receiver adj	0(0)	2(0)	5(0)	8(0)	10(0)	11(1)
+20	Payer adj	27(1)	25(1)	23(1)	20(1)	16(2)	13(1)
	Receiver adj	0(0)	0(0)	1(0)	2(0)	2(0)	4(0)
+60	Payer adj	80(4)	82(4)	67(4)	64(4)	55(3)	48(3)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
+90	Payer adj	87(6)	86(6)	88(6)	78(5)	80(5)	71(4)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
+99	Payer adj	10(2)	21(3)	52(5)	68(5)	73(5)	76(5)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)


100
80
60
40
20
0
−100
−50
0
50
100
counterparty adjustment (bps)
correlation (%)
Payer
Receiver
$\nu_1 = 0.10$
$\nu_1 = 0.50$

Counterparty

Counterparty Risk Engine

Underlying

Model for the Counterparty default time:
CIR++ calibrated on the credit spreads of the counterparty

Model for the Underlying:
Interest rates: G2++
Commodity: Schwart-Smith + Shift.
Credit: CIR++

τ_C

Correlation

$[\mathrm{NPV}(\tau_C)]^+$

$$\mathrm{E}_t\left\{\Pi^D(t,T)\right\} = \mathrm{E}_t\left\{\Pi(t,T)\right\} - \mathrm{LGD}\cdot\mathrm{E}_t\left\{\mathbf{1}(t<\tau_C\leqslant T)\cdot D(t,\tau_C)\cdot[\mathrm{NPV}(\tau_C)]^+\right\}$$

Conclusions

- Counterparty Risk adds one level of optionality.
- Induced model dependence raises issues already at underlying model level.
- Analysis including underlying asset/ counterparty default correlation requires a credit model.
- Highly specialized hybrid modeling framework.
- Accurate arbitrage-free valuation.
- Accurate scenarios for wrong way risk.

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