Exercises, November 15 2005

Exercise 1

Consider the following risk neutral dynamics with shocks in dimension three, where Z_1, Z_2 and Z_3 are standard Brownian motions, correlated according to a constant c,
i.e. $dZ_i dZ_j = c\ dt$, and where K is a positive constant equal to the risk free interest rate:

$$\begin{aligned} dX_t &= K X_t dt + \sigma_1 X_t dZ_1(t) \\ dY_t &= \mu_Y(t, Y_t)dt + \sigma_2 \sqrt{Y_t} dZ_2(t) \\ dP_t &= \mu_P(t, P_t)dt + \sigma_3 P_t^2 dZ_3(t). \end{aligned}$$

1.1) Compute DC(X_t), DC($\ln(X_t)$), DC(Y_t), DC((Y_t^2)), DC(P_t), DC($P_t + \ln(Y_t)$), DC($\ln(P_t) + Y_t^2$).

1.2) Compute the dynamics of P under the measure Q^X, i.e. under the measure having X as numeraire (is X a possible numeraire?)

Exercise 2.

a. Consider a caplet with first reset date $T_1 = 1y6m$, and with maturity $T_2 = 2y$. Let the volatility of the underlying forward rate be constant and equal to 20%. Given the two bond prices $P(0, 1y6m) = 0.9898$, $P(0, 2y) = 0.98$, compute the 1y6m-2y caplet price for the strike $K = 0.02$.

b. Consider the same caplet as before but this time let the instantaneous volatility of the underlying forward rate at time t be $\sigma_2(t) = 0.01t + 0.00025$ instead of 20%. Compute the caplet price.

Exercise 3. In the LIBOR market model, write the dynamics of $F_3 = F(\cdot, T_2, T_3)$ under Q^2 and the dynamics of $F_2 = F(\cdot, T_1, T_2)$ under Q^3. Do all these dynamics depend on the correlation between interest rates?